Functions

Function Notation:

A function can be thought of as a Machine that assigns 1 output to every input.

Name of the function

Inside the ( ) is the input of the func. This is what has been or should be plugged into the function

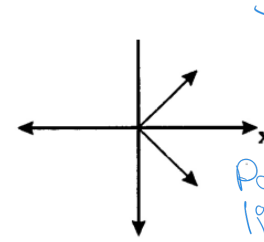
$$\underbrace{f(x)} = \underbrace{3x + 4}$$

read as "f of x"

f(x) is the output of the function f when the input is x

Rule used by the function machine.

Examples:

1. If $f(x) = 3x^2 - 4x$, find the function value for: a. $f(6) = 3(6)^2 - 4(6)$ $3(36) - 24 = \boxed{84}$ b. $f(x+2) = 3(x+2)^2 - 4(x+2)$ $3(x^2+4x+4) - 4x - 8$ $3x^2 + 12x + 12 - 4x - 8 = \boxed{3x^2 + 8x + 4}$	2. If $f(x) = 6$, find the function value for: a. $f(-3) = 6$ b. $f(x^2) = 6$
3. If $f(x) = \begin{cases} \lvert x+2 \rvert, x \le 2 \\ \frac{2}{3}x - 1, 2 < x \le 6 \\ 3, x > 6 \end{cases}$ find the function value for: a. $f(6) = \frac{2}{3}(6) - 1 = \boxed{3}$ b. $f(0) = \lvert 0 + 2 \rvert = \boxed{2}$	4. Determine with the relation is a function: a. <table><tr><td>x</td><td>–1</td><td>2</td><td>5</td></tr><tr><td>y</td><td>–4</td><td>8</td><td>20</td></tr></table> $-1 - -4$ $2 - 8$ $5 - 12$ yes b.  No, does not pass the vertical line test c. {-1, 5), (2, 5), (3, 5) (8, 5)} $-1, 2, 3, 8 \to 5$ yes

Function Operations:

Operations with Functions	
$(f+g)(x)$ means $f(x)+g(x)$ *Example:* $f(x)=3x+5, g(x)=2x-7$ $3x+5+2x-7$ $\boxed{5x-2}$	$(f-g)(x)$ means $f(x)-g(x)$ *Example: :* $f(x)=2x-4, g(x)=x+3$ $2x-4-(x+3)$ $2x-4-x-3$ $\boxed{x-7}$
$(f\cdot g)(x)$ means $f(x)\cdot g(x)$ *Example: :* $f(x)=x+2, g(x)=x-3$ $(x+2)(x-3)$ $\boxed{x^2-x-6}$	$\frac{f}{g}(x)$ means $\frac{f(x)}{g(x)}$ *Example: :* $f(x)=x^2-16, g(x)=x+4$ $\frac{x^2-16}{x+4}=\frac{\cancel{(x+4)}(x-4)}{\cancel{(x+4)}}=\boxed{x-4}$

Composition of functions:

Example: **f(x) = 2x+3** and **g(x) = x^2**

"x" is just a placeholder, and to avoid confusion let's just call it "input":

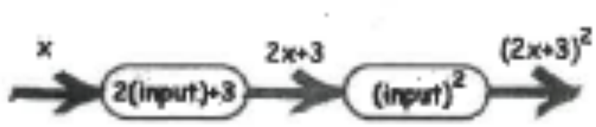
f(input) = 2(input)+3

g(input) = (input)^2

So, let's start:

(g ∘ f)(x) = g(f(x))

First we apply **f**, then apply **g** to that result:



(g ∘ f)(x) = (2x+3)^2

Examples: Let $f(x)=x^2, g(x)=\sqrt{x}$+1, h(x)=2x+3

1. $fog(4)=f(g(4))=f(\sqrt{4}+1)$ $=f(2+1)=f(3)=3^2=\boxed{9}$
2. $g(h(x))=g(2x+3)$ $=\boxed{\sqrt{2x+3}+1}$
3. $f(h(x))=f(2x+3)=(2x+3)^2$ $=\boxed{4x^2+12x+9}$

Inverse Relations: "switch the x and y", if the inverse is a function then it is called an inverse function

Examples:

1. Find the inverse realation from the table:

x	0	1	2	3	4
y	3	5	7	9	11

x	3	5	7	9	11
y	0	1	2	3	4

Is the inverse a function?
yes

2. Find the equation of the inverse relation

$y = \frac{1}{2}x + 4$

$x = \frac{1}{2}y + 4$

$(x - 4 = \frac{1}{2}y)2$

$2x - 8 = y^{-1}$

$\boxed{f^{-1}(x) = 2x - 8}$

Is the inverse a function? yes

3. Verify the *f* and *g* are inverses:

$f(x) = x + 2;\ g(x) = x - 2$

$f(g(x)) = x - 2 + 2 = x$ ✓

$g(f(x)) = x + 2 - 2 = x$

yes

Rates of change: Simply means slope

Slope of any line by using the slope formula between 2 points.

"Average Rate of Change" Simply means draw a line through 2 points and find the slope.

Examples:

1. Find the average rate of change of $f(x) = x^2 + 3$ on the interval [0, 2]

$(0, 3)(2, 7)$

$f(0) = 3$

$f(2) = 2^2 + 3 = 7$

Averoc $= \frac{7-3}{2-0} = \frac{4}{2} = \boxed{2}$

Find the average rate of change of $f(x) = 3x^3$on the interval [-2, 3]

$f(-2) = 3(-8) = -24$

$f(3) = 3(27) = 81$

Aroc $= \frac{81+24}{3+2} = \frac{105}{6} = \boxed{17.5}$