Integration Techniques

Name key

Date ______________

Assignment 8.3b

Find each integral:

1.

$\int cosx \cdot ln(sinx)dx = \int \ln a \, da = a \cdot \ln a - \int a \cdot \frac{1}{a} da = a\ln a - a + C =$

$\sin x = a$

$\cos x \, dx = da$

$\ln a = u$

$\frac{1}{a} da = du$

$dv = da$

$v = a$

$= \boxed{\sin x \cdot \ln|\sin x| - \sin x + C}$

2.

$\int \sqrt{x} \ln x dx = \frac{2}{3}x^{\frac{3}{2}} \ln x - \int \frac{1}{x} \cdot \frac{2}{3} x^{\frac{3}{2}} dx =$

$\ln x = u$

$\frac{1}{x} dx = du$

$dv = x^{\frac{1}{2}} dx$

$v = \frac{2}{3} x^{\frac{3}{2}}$

$= \boxed{\frac{2}{3} x^{\frac{3}{2}} \ln x - \frac{4}{9} x^{\frac{3}{2}} + C}$

3.

$\int \cos(2x) \cdot e^{-x} dx = -e^{-x}\cos 2x - \int 2e^{-x} \sin 2x dx = -e^{-x}\cos 2x - 2\left(-e^{-x}\sin 2x + 2\int e^{-x}\cos 2x \, dx\right.$

$\cos 2x = u$

$-2\sin 2x dx = du$

$dv = e^{-x} dx$

$v = -e^{-x}$

$\sin 2x = u$

$2 \cdot \cos 2x dx = du$

$dv = e^{-x} dx$

$v = -e^{-x}$

$5\int e^{-x} \cos 2x dx = -e^{-x}\cos 2x + 2e^{-x}\sin 2x + C$

$\boxed{\int e^{-x} \cos 2x dx = -\frac{1}{5}e^{-x}\cos 2x + \frac{2}{5}e^{-x}\sin 2x + C}$

4.

$\int \ln(x+3) \, dx$

$\ln(x+3) = u$

$\frac{1}{x+3} dx = du$

$dx = dv$

$x = v$

$\int \ln|x+3| dx = x \cdot \ln|x+3| - \int \frac{x}{x+3} dx =$

$x \cdot \ln|x+3| - \int\left(1 - \frac{3}{x+3}\right) dx =$

$\begin{array}{r} 1 \\ x+3 \overline{) x } \\ \underline{x+3} \\ -3 \end{array}$

$\boxed{x \ln|x+3| - x + 3\ln|x+3| + C}$

sign	u/du	dv/v
+	x^2	e^{-2x}
–	$2x$	$-\frac{1}{2}e^{-2x}$
+	2	$+\frac{1}{4}e^{-2x}$
–	0	$-\frac{1}{8}e^{-2x}$

5.
$$\int \frac{x^2}{e^{2x}}dx$$

$$\int \frac{x^2}{e^{2x}}\,dx = \boxed{-\frac{1}{2}e^{-2x}\cdot x^2 - \frac{1}{2}x\cdot e^{-2x} - \frac{1}{4}e^{-2x} + C}$$

6.
$$\int x\cdot 2^x dx \quad = \frac{x}{\ln 2}\cdot 2^x - \int \frac{1}{\ln 2}\, 2^x\,dx = \boxed{\frac{x}{\ln 2}\cdot 2^x - \left(\frac{1}{\ln 2}\right)^2\cdot 2^x + C}$$

$u = x \qquad dv = 2^x\,dx$

$du = dx \qquad v = \frac{1}{\ln 2}\cdot 2^x$

7.
$$\int_0^{\frac{\pi}{2}} \sin^2 x\cdot\cos^3 x\,dx = \int_0^{\frac{\pi}{2}} \sin^2 x\cdot(1-\sin^2 x)\cos x\,dx = \int_0^1 u^2 - u^4\,du = \left(\frac{1}{3}u^3 - \frac{1}{5}u^5\right)\Big|_0^1 =$$

$\sin x = u \qquad \sin 0 = 0$

$\cos x\,dx = du \qquad \sin\frac{\pi}{2} = 1$

$$= \frac{1}{3} - \frac{1}{5} = \boxed{\frac{2}{15}}$$

8.
$$\int \cos^3(2-x)\cdot\sin(2-x)\,dx = \int u^3\,du = \boxed{\frac{1}{4}\cdot\cos^4(2-x) + C}$$

$\cos(2-x) = u$

$-\sin(2-x)\cdot(-1)\,dx = du$

9.
$$\int \sec^3 x\,dx = \int (1+\tan^2 x)\sec x\,dx = \int \sec x\,dx + \int \tan^2 x\sec x\,dx$$

$u = \tan x \qquad dv = \tan x\sec x\,dx$

$du = \sec^2 x\,dx \qquad v = \sec x$

$$\int \sec^3 x\,dx = \ln|\sec x + \tan x| + \sec x\tan x - \int \sec x\cdot\sec^2 x\,dx$$

$$2\int \sec^3 x\,dx = \ln|\sec x + \tan x| + \sec x\tan x + C$$

$$\int \sec^3 x\,dx = \boxed{\frac{1}{2}\ln|\sec x + \tan x| + \frac{1}{2}\sec x\tan x + C}$$

$t^2 = a$
$2t\,dt = da$

$\sin a = u$
$\cos a\, da = du$

10.
$$\int t\cdot\cos^3(t^2)\,dt = \frac{1}{2}\int \cos^3 a\, da = \frac{1}{2}\int (1-\sin^2 a)\cos a\, da =$$
$$= \frac{1}{2}\int (1-u^2)\,du = \frac{1}{2}\sin a - \frac{1}{6}\sin^3 a + C =$$
$$= \boxed{\frac{1}{2}\sin t^2 - \frac{1}{6}\sin^3 t^2 + C}$$

11.
$$\int \sin^2 x\cdot\cos^4 x\,dx = \int (\cos^4 x - \cos^6 x)\,dx = \int \left(\frac{1+\cos 2x}{2}\right)^2 - \left(\frac{1+\cos 2x}{2}\right)^3 dx =$$

$\sin^2 x = 1-\cos^2 x$

$$\int \frac{1}{4}(1+2\cos 2x+\cos^2 2x) - \frac{1}{8}(1+\cos 2x+\cos^2 2x+\cos^3 2x)\,dx =$$

$\sin 2x = u$
$2\cos 2x = du$

$$\int \frac{1}{4}\left(1+2\cos 2x+\frac{1+\cos 4x}{2}\right) - \frac{1}{8}\left(1+\cos 2x+\frac{1+\cos 4x}{2}+(1-\sin^2 2x)\cos 2x\right)dx =$$
$$= \frac{1}{4}x + \frac{1}{4}\sin 2x + \frac{1}{8}x + \frac{1}{32}\sin 4x - \frac{1}{8}x - \frac{1}{16}\sin 2x - \frac{1}{16}x - \frac{1}{64}\sin 4x - \frac{1}{16}\sin 2x - \frac{1}{48}\sin^3 2x + C$$
$$= \boxed{\frac{3}{16}x + \frac{1}{8}\sin 2x + \frac{1}{64}\sin 4x - \frac{1}{48}\sin^3 2x + C}$$

12.
$$\int \sin(2x)\cdot\cos(2x)\,dx$$

$\sin(2x) = u$
$2\cos(2x)\,dx = du$

$$\int \sin(2x)\cdot\cos(2x)\,dx = \frac{1}{2}\int u\,du = \boxed{\frac{1}{4}\cdot\sin^2(2x) + C}$$

13. Use the substitution $u = \sqrt{x}$ to rewrite $\int_0^{\pi} \cos\sqrt{x}\,dx$. $= \int_0^{\sqrt{\pi}} \cos u\cdot 2u\,du$

$u = \sqrt{x}$ $\quad u^2 = x$ $\quad x = 0$
$du = \frac{1}{2}\cdot\frac{1}{\sqrt{x}}dx$ $\quad u = 0$
$x = \pi$
$u = \sqrt{\pi}$
$dx = 2\sqrt{x}\cdot du$
$dx = 2u\,du$

14.
$$\int x^3\sin(x^2)\,dx = -\frac{1}{2}x^2\cos(x^2) + \int \frac{1}{2}\cdot 2x\cos(x^2)\,dx =$$
$$= \boxed{-\frac{1}{2}x^2\cos(x^2) + \frac{1}{2}\sin(x^2) + C}$$

$u = x^2$
$du = 2x\,dx$
$dv = x\sin(x^2)\,dx$
$v = -\frac{1}{2}\cos(x^2)$

15.

$$\int \sin 8x \cos 5x \, dx = \int \frac{1}{2}\left(\sin 13x + \sin 3x\right)dx =$$

$$= \frac{1}{2}\cdot\frac{1}{13}\cdot\left(-\cos(13x)\right) + \frac{1}{2}\cdot\frac{1}{3}\cdot\left(-\cos(3x)\right)\Big) + C$$

$$= \boxed{-\frac{1}{26}\cos(13x) - \frac{1}{6}\cos(3x) + C}$$

16.

$$\int \frac{1-\tan^2 x}{\sec^2 x}\,dx = \int \frac{1-(\sec^2 x - 1)}{\sec^2 x}\,dx = \int \frac{2-\sec^2 x}{\sec^2 x}\,dx =$$

$$= \int\left(\frac{2}{\sec^2 x} - 1\right)dx = \int\left(2\cdot\cos^2 x - 1\right)dx =$$

$$\int\left(2\cdot\frac{1}{2}\left(1+\cos 2x\right) - 1\right)dx =$$

$$\int \cos 2x \, dx = \boxed{\frac{1}{2}\sin 2x + C}$$