

## Review Chapter 5 "Polynomial Functions"

Name \_\_\_\_\_

*key*

Date \_\_\_\_\_

### Assignment 5R

Write each number in scientific notation:

$$1. (4.3 \times 10^4)^{-2}$$

$$(4.3)^{-2} \times 10^{-8} =$$

$$0.0541 \times 10^{-8} =$$

$$5.41 \cdot 10^{-2} \times 10^{-8} = \boxed{5.41 \times 10^{-10}}$$

$$2. 2.5 \times 10^4 \cdot 6.5 \times 10^{-2}$$

$$2.5 \cdot 6.5 \times 10^{4+(-2)} =$$

$$16.25 \times 10^2 =$$

$$1.625 \cdot 10^1 \times 10^2 = \boxed{1.625 \times 10^3}$$

$$3. \frac{2.5 \times 10^2}{5.7 \times 10^4}$$

$$\frac{2.5}{5.7} \times \frac{10^2}{10^4} =$$

$$0.439 \times 10^{-2}$$

$$4.39 \cdot 10^{-1} \times 10^{-2} = \boxed{4.39 \times 10^{-3}}$$

Evaluate/Simplify each expression

$$4. 5^5 \cdot 5^{-2} = 5^{5+(-2)} = \boxed{5^3 = 125}$$

$$5. (7^4)^2 = \boxed{7^8}$$

$$6. \left(\frac{2^{-2}}{3}\right)^2 = \frac{2^{-4}}{3^2} =$$

$$= \frac{1}{3^2 \cdot 2^4} = \frac{1}{9 \cdot 16} = \boxed{\frac{1}{144}}$$

$$7. \left(\frac{3}{4}\right)^{-3} = \frac{3^{-3}}{4^{-3}} = \frac{4^3}{3^3} = \boxed{\frac{64}{27}}$$

$$8. (a^5 b^{-7})(a^{-4} b^9)$$

$$a^{5+(-4)} \cdot b^{-7+9} = \boxed{a^1 b^2}$$

$$9. (r^{-3} s^4)^{-4}$$

$$r^{12} s^{-16} = \boxed{\frac{r^{12}}{s^{16}}}$$

10. Use direct substitution to evaluate  $-5x^3 - x^2 + 6x - 4$  for  $x = -1$ .

$$-5 \cdot (-1)^3 - (-1)^2 + 6 \cdot (-1) - 4 = 5 - 1 - 6 - 4 = \boxed{-6}$$

11. Use synthetic substitution to evaluate  $2x^4 + x^2 - 4x + 10$  for  $x = 3$ .

$$2x^4 + 0x^3 + x^2 - 4x + 10$$

$$\begin{array}{r|rrrrr} 3 & 2 & 0 & 1 & -4 & 10 \\ & & 3 \cdot 2 & 3 \cdot 6 & 3 \cdot 19 & 3 \cdot 53 \\ \hline & 2 & 6 & 19 & 53 & \boxed{169} \end{array}$$

Perform the indicated operations:

$$12. (2x^4 + 9x - 7) - (x^4 + 6x + 5)$$

$$- x^4 + 6x + 5$$

$$x^4 + 3x - 12$$

$$13. (x-6)(x^2-8x+9)$$

$$x^2 - 8x + 9$$

|    |         |         |       |
|----|---------|---------|-------|
| x  | $x^3$   | $-8x^2$ | $9x$  |
| -6 | $-6x^2$ | $48x$   | $-54$ |

$$= x^3 - 14x^2 + 57x - 54$$

$$14. (x-2)(x+3)(x-5) = (x^2+x-6)(x-5) = x^3 - 4x^2 - 11x + 30$$

$$x^2 + x - 6$$

|    |         |       |       |
|----|---------|-------|-------|
| x  | $x^3$   | $x^2$ | $-6x$ |
| -5 | $-5x^2$ | $-5x$ | $+30$ |

$$15. (7x-3)^2 = (7x-3)(7x-3) = 49x^2 - 21x - 21x + 9$$

$$= 49x^2 - 42x + 9$$

$$16. x^4 - 81 = (x^2)^2 - (9)^2$$

Factor the polynomial completely

$$= (x^2 - 9)(x^2 + 9) = (x^2 - 3^2)(x^2 + 9)$$

$$(x-3)(x+3)(x^2+9)$$

$$17. y^3 + 6y^2 - 9y - 54 = y^2(y+6) - 9(y+6) =$$

$$(y+6)(y^2-9) = (y+6)(y-3)(y+3)$$

Divide using polynomial long division or synthetic method.

18.  $(3x^3 + 11x^2 + 4x + 1) \div (x^2 + x)$

long division

$$\begin{array}{r} 3x + 8 \\ x^2 + x \overline{) 3x^3 + 11x^2 + 4x + 1} \\ \underline{3x^3 + 3x^2} \phantom{+ 1} \\ 8x^2 + 4x + 1 \\ \underline{8x^2 + 8x} \phantom{+ 1} \\ -4x + 1 \end{array}$$

Answer:  $3x + 8 + \frac{-4x + 1}{x^2 + x}$

19.  $(x^4 - 5x^3 - 8x^2 + 13x - 12) \div (x - 6)$

long division.  $x^3 + x^2 - 2x + 1$

$$\begin{array}{r} x - 6 \overline{) x^4 - 5x^3 - 8x^2 + 13x - 12} \\ \underline{-x^4 + 6x^3} \phantom{- 8x^2 + 13x - 12} \\ -x^3 - 8x^2 + 13x - 12 \\ \underline{-x^3 + 6x^2} \phantom{+ 13x - 12} \\ -2x^2 + 13x - 12 \\ \underline{-2x^2 + 12x} \phantom{- 12} \\ -x - 12 \\ \underline{x - 6} \\ -6 \end{array}$$

synthetic:

$$\begin{array}{r|rrrrrr} 6 & 1 & -5 & -8 & 13 & -12 \\ & & +6 & +1 & +6 & +6 \\ \hline & 1 & 1 & -2 & 1 & -6 \end{array}$$

answer:  $x^3 + x^2 - 2x + 1 + \frac{-6}{x - 6}$

20.  $(x^4 + 10x^3 + 8x^2 - 59x + 40) \div (x^2 + 3x - 5)$

long division

$$\begin{array}{r} x^2 + 7x - 8 \\ x^2 + 3x - 5 \overline{) x^4 + 10x^3 + 8x^2 - 59x + 40} \\ \underline{-x^4 + 3x^3 - 5x^2} \phantom{- 59x + 40} \\ 7x^3 + 13x^2 - 59x + 40 \\ \underline{7x^3 + 21x^2 - 35x} \phantom{+ 40} \\ -8x^2 - 24x + 40 \\ \underline{-8x^2 - 24x + 40} \\ 0 \end{array}$$

answer:  $x^2 + 7x - 8$

21.  $(x^3 - 13x - 12) \div (x - 4)$

long division

$$\begin{array}{r} x^2 + 4x + 3 \\ x - 4 \overline{) x^3 + 0x^2 - 13x - 12} \\ \underline{-x^3 + 4x^2} \phantom{- 13x - 12} \\ 4x^2 - 13x - 12 \\ \underline{4x^2 - 16x} \phantom{- 12} \\ 3x - 12 \\ \underline{3x - 12} \\ 0 \end{array}$$

synthetic:

$$\begin{array}{r|rrrr} 4 & 1 & 0 & -13 & -12 \\ & & +4 & +4 & +3 \\ \hline & 1 & 4 & 3 & 0 \end{array}$$

answer:  $x^2 + 4x + 3$

22. Given polynomial  $f(x)$  and a factor of  $f(x)$ , factor  $f(x)$  completely.

$$f(x) = x^3 - 10x^2 + 19x + 30, \quad x - 6$$

$$\begin{array}{r|rrrr} 6 & 1 & -10 & 19 & 30 \\ & & +6 \cdot 1 & +6 \cdot -4 & +6 \cdot -5 \\ \hline & 1 & -4 & -5 & 0 \end{array}$$

$$x^2 - 4x - 5 = (x - 5)(x + 1)$$

$$f(x) = (x - 6)(x - 5)(x + 1)$$

23. Given polynomial function  $f(x)$  and a zero of  $f(x)$ , find other zeroes.

$$f(x) = x^3 - 2x^2 - 21x - 18, \quad -3 \quad x = -3$$

$$\begin{array}{r|rrrr} -3 & 1 & -2 & -21 & -18 \\ & & +3 \cdot 1 & +3 \cdot -5 & +3 \cdot -6 \\ \hline & 1 & -5 & -6 & 0 \end{array}$$

$$x^2 - 5x - 6 = (x - 6)(x + 1)$$

$$0 = (x + 3)(x - 6)(x + 1)$$

$$x = -3 \quad x = 6 \quad x = -1$$

Find the real-number solutions of each equation.

24.  $18x^3 = 50x$

$$18x^3 - 50x = 0$$

$$2x(9x^2 - 25) = 0$$

$$2x(3x - 5)(3x + 5) = 0$$

$$x = 0 \quad x = \frac{5}{3} \quad x = -\frac{5}{3}$$

25.  $3x^4 - 27x^2 + 9x = x^3$

$$3x^4 - x^3 - 27x^2 + 9x = 0$$

$$x^3(3x - 1) - 9x(3x - 1) = 0$$

$$(x^3 - 9x)(3x - 1) = 0$$

$$x(x^2 - 9)(3x - 1) = 0$$

$$x(x - 3)(x + 3)(3x - 1) = 0$$

$$x = 0 \quad x = 3 \quad x = -3 \quad x = \frac{1}{3}$$

26. Simplify  $\frac{2b^{-2}a^3}{(2a^2b^{-3})^{-3}} = \frac{2a^3}{b^2 \cdot 2^{-3} \cdot a^{-6} b^9} = \frac{2a^3 \cdot 2^3 a^6}{b^2 \cdot b^9} =$   
 $= \frac{2^4 a^9}{b^{11}} = \boxed{\frac{16a^9}{b^{11}}}$

27. Factor completely  $27a^3 - 64$   
 $(3a)^3 - 4^3 = (3a - 4)((3a)^2 + 3a \cdot 4 + 4^2)$   
 $(3a - 4)(9a^2 + 12a + 16)$

28. Write a polynomial function  $f$  of least degree that has rational coefficients, a leading coefficient of 1, and the zeros 1, -4, and -2i.

$$\begin{array}{cccc} x=1 & x=-4 & x=-2i & x=2i \\ x-1=0 & x+4=0 & x+2i=0 & x-2i=0 \end{array}$$

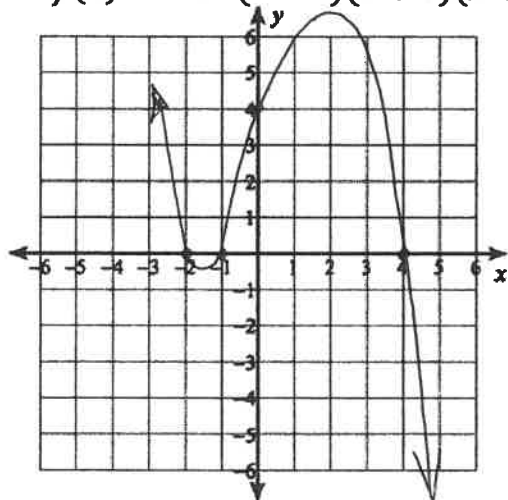
$$\begin{aligned} f(x) &= (x-1)(x+4)(x+2i)(x-2i) \\ &= (x^2+3x-4)(x^2-2ix+2ix-4i^2) \\ &= (x^2+3x-4)(x^2+4) = \boxed{x^4+3x^3+12x-16} \end{aligned}$$

$$\begin{array}{r} x^2+3x-4 \\ x^2 \begin{array}{|c|c|c|} \hline x^4 & 3x^3 & -4x^2 \\ \hline \end{array} \\ +4 \begin{array}{|c|c|c|} \hline 4x^2 & 12x & -16 \\ \hline \end{array} \end{array}$$

Sketch graph of each function.

29.

$$f(x) = -0.5(x-4)(x+2)(x+1)$$



degree = 3  
negative leading  
coefficient



x-intercepts

$$x = 4$$

$$x = -2$$

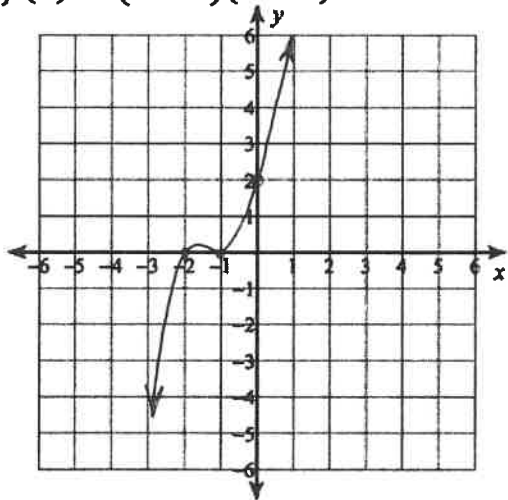
$$x = -1$$

y-intercept:

$$x = 0 \quad y = -0.5(0-4)(0+2)(0+1)$$
$$= -0.5 \cdot (-4) \cdot 2 \cdot 1 = 4$$

30.

$$f(x) = (x+2)(x+1)^2$$



degree = 3  
positive  
leading coefficient



x-intercepts

$$x = -2$$

$$x = -1$$

$$x = -1$$

y-intercept

$$x = 0$$
$$y = (0+2)(0+1)^2 = 2$$